1

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Find a function f such that $\nabla f = \vec{F}$ where $\vec{F}(x,y) = xy^2\,\hat{i} + x^2y\,\hat{j}$ and use it to compute $\int_C \vec{F}\cdot d\vec{r}$ where C is the line segment from $(0,0)$ to $(2,2)$.

Soln $\nabla f = \vec{F} \Rightarrow \langle f_x(x,y), f_y(x,y)\rangle = \langle xy^2, x^2y\rangle$

So, $f_x(x,y) = xy^2 \Rightarrow f(x,y) = \frac{1}{2}x^2y^2 + g(y)$

Then, $f_y(x,y) = \frac{1}{2}xy^2 + g'(y)$

Since $f_y(x,y) = x^2y$, we have $x^2y = x^2y + g'(y) \Rightarrow g'(y) = 0 \Rightarrow g(y) = K$

Therefore, $f(x,y) = \frac{1}{2}x^2y^2 + K$ and we can pick $K = 0$, so $f(x,y) = \frac{1}{2}x^2y^2$

Then, $\int_C \vec{F}\cdot d\vec{r} = \int_C \nabla f\cdot d\vec{r} = f(2,2) - f(0,0) = \frac{1}{2}(2)^2(2)^2 - 0 = 8$